# On the Hydromagnetic Kelvin-Helmholtz Instability between Compressible Fluids

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We have discussed the effect of gravity on the hydromagnetic Kelvin-Helmholtz instability of a plane interface between compressible, inviscid, infinitely conducting fluids. The stability of the interface is investigated including gravity. The solar plasma and the magnetospheric medium are supposed to be of equal density and to carry a uniform magnetic field ($\boldsymbol{H}$) in the direction of streaming. The cases (i) $\boldsymbol{H}_1 \neq \boldsymbol{H}_2$ and $\varkappa_1$ ($\varkappa = c_p/c_v$) not necessarily equal to $\varkappa_2$, (ii) $\boldsymbol{H}_1 = \boldsymbol{H}_2$, $\varkappa_1 \neq \varkappa_2$ and (iii) $\boldsymbol{H}_1 = \boldsymbol{H}_2$, $\varkappa_1 = \varkappa_2$ are discussed for perturbations, transverse as well as parallel to the direction of streaming. It is concluded that the interface is unstable in all the cases except for transverse perturbations, the two media carrying the same magnetic field and being characterized by the same $\varkappa$, when it is found to be stable for disturbances parallel to the streaming verlocity.

## 1. Introduction

The study of the stability of the interface between the solar wind and the magnetosphere has been a subject of great controversy. In a theoretical study Dungey [1] and Parker [2, 3] conclude that the magnetospheric boundary is unstable. Assuming the magnetic field in the solar wind to be zero, Dungey [1] has discussed the effect of perfect compressibility of the magnetospheric plasma and concluded that it is negligible. Parker has also neglected the magnetic field in the solar wind. Sen [4] studies the effect of the magnetic field and compressibility and conludes that they have a profound effect on the stability. Dessler [5] contends that the interface is stable. If this inference is correct, the theories of the Aurora, magnetic storms and Van Allen radiation dependent on the concept of turbulent solar injection need to be re-examined.

Fejer [6] has discussed the hydromagnetic reflection and refraction at a fluid velocity discontinuity. Fejer [7] has also discussed the hydromagnetic stability of a velocity discontinuity at a plane interface between two perfectly conducting, inviscid, compressible fluids. Lerche [8] has discussed the validity of the hydromagnetic approach in discussing instability of the magnetospheric boundary.

Talwar [9] discusses the stability of the magnetospheric boundary, without including gravity, viewing the interface as a surface of Kelvin-Helmholtz discontinuity. However, the dispersion relation obtained by him is incorrect rendering his conclusions unreliable.

Southwood [10] has discussed the hydromagnetic Kelvin-Helmholtz stability of a plane interface between compressible infinitely conducting fluids. The critical relative velocity for stability has been discussed. Mekenzie [11] has applied the results of the analysis of hydromagnetic reflection and refraction at a shear layer and at a shock in situations representative of the magnetopause and the Earth's bow shock.

In this paper we wish to investigate the hydromagnetic stability of the interface between two compressible fluids including gravity. We consider the media as plasmas of zero dissipation and uniform density, being infinitely extended and having a planar interface. We also assume that both fluids carry homogeneous constant magnetic fields which may be unequal, leading to a current sheet at the interface.

## 2. Basic Equations

Consider a system of cartesian axes with the $z$ direction vertical. Let the uniform velocities and the uniform magnetic fields, both in the horizontal plane, be $\boldsymbol{U}_1$, $\boldsymbol{U}_2$ and $\boldsymbol{H}_1$, $\boldsymbol{H}_2$ respectively for $z<0$ and $z>0$. Let $\varrho_1$ and $\varrho_2$ denote the uniform densities of the lower and the upper plasmas char-

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acterized by the sound speeds $c_1$ and $c_2$ respectively. In the presence of gravity, the steady state of the system requires

$$p_d + \tfrac{1}{2} H^2 + g \varrho z = \text{constant} \quad (1)$$

for either medium, where $p_d$ is the dynamic pressure. The equilibrium of the interface ($z = 0$) is governed by

$$(p_d)_1 + \tfrac{1}{2} H_1^2 = (p_d)_2 + \tfrac{1}{2} H_2^2 . \quad (2)$$

Making use of the normal mode analysis we put

$$\boldsymbol{v} = \boldsymbol{U}_0 + \boldsymbol{u}, \quad \varrho = \varrho_0 + \delta\varrho, \quad p_d = p_0 + \delta p, \quad (3)$$
$$\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{h}, \boldsymbol{U}_0 = (U_{0x}, U_{0y}, 0), \boldsymbol{H}_0 = (H_{0x}, H_{0y}, 0),$$

where $\boldsymbol{u}$, $\delta\varrho$, $\delta p$ and $\boldsymbol{h}$ denote perturbations in velocity, density, dynamic pressure and magnetic field respectively. The suffix 0 refers to equilibrium values.

Assuming the perturbations to vary with $x$, $y$, $z$ and $t$ as

$$f(z) \exp\{i k_x x + i k_y y + n t\} \quad (4)$$

and making use of linearised hydromagnetic equations we obtain the following equations

$$\varrho_0 (n + i \boldsymbol{k} \cdot \boldsymbol{U}_0) (\nabla \cdot \boldsymbol{u} - D w) + (i k_x h_y - i k_y h_x) (i k_x H_{0y} - i k_y H_{0x}) = k^2 \delta p , \quad (5)$$

$$\varrho_0 (n + i \boldsymbol{k} \cdot \boldsymbol{U}_0) w = - (D + g/c^2) \ \delta p - H_{0x} (D h_x - i k_x h_z) + H_{0y} (i k_y h_z - D h_y) , \quad (6)$$

where $k^2 = k_x^2 + k_y^2$, $\mathrm{D} \equiv \mathrm{d}/\mathrm{d}z$ and $w$ is the $z$-component of velocity. Eliminating $\delta p$ from (5) and (6), we obtain

$$\begin{aligned} &- \varrho_0 (n + i \boldsymbol{k} \cdot \boldsymbol{U}_0) \ [(D + g/c^2) D - k^2] w + \varrho_0 (n + i \boldsymbol{k} \cdot \boldsymbol{U}_0) \ (D + g/c^2) \nabla \cdot \boldsymbol{u} \\ &+ H_{0y} [i k_x (D + g/c^2) \ (i k_x h_y - i k_y h_x) + k^2 (D h_y - i k_y h_z)] \\ &- H_{0x} [i k_y (D + g/c^2) \ (i k_x h_y - i k_y h_x) - k^2 (D h_x - i k_x h_z)] = 0 . \end{aligned} \quad (7)$$

The magnetic field term in Eq. (7) simplifies to

$$- \frac{(\boldsymbol{H}_0 \cdot \boldsymbol{k})^2}{(n - i \boldsymbol{k} \cdot \boldsymbol{U}_0)} (D^2 - k^2) w + \frac{1}{(n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)} \frac{g}{c^2} [(\boldsymbol{H}_0 \cdot \boldsymbol{k}) (\boldsymbol{H}_0 \times \boldsymbol{k}) \cdot \boldsymbol{\zeta} + (\boldsymbol{H}_0 \times \boldsymbol{k}) (\boldsymbol{H}_0 \times \boldsymbol{k})^2 \nabla \cdot \boldsymbol{u}] \quad (8)$$

where $\boldsymbol{\zeta} = \boldsymbol{z} (i k_x v - i k_y u)$. $\boldsymbol{z}$ is a unit vector along the $z$-axis. Combining Eq. (7) and (8), we have

$$\begin{aligned} &\left[1 + \frac{(\boldsymbol{k} \cdot \boldsymbol{V}_0)^2}{(n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2}\right] (D^2 - k^2) w + \frac{g}{c^2} D w - \left[D + \frac{g}{c^2} \left\{1 + \frac{(\boldsymbol{k} \times \boldsymbol{U}_0)^2}{(n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2}\right\}\right] \nabla \cdot \boldsymbol{u} \\ &+ \frac{g}{c^2 (n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2} (\boldsymbol{k} \cdot \boldsymbol{V}_0) (\boldsymbol{k} \times \boldsymbol{V}_0) \cdot \boldsymbol{\zeta} = 0 , \end{aligned} \quad (9)$$

where $\boldsymbol{V}_0 (= \boldsymbol{H}_0 / \varrho_0^{1/2})$ denotes the Alfvén velocity vector. Following Talwar (1964) to eliminate $\boldsymbol{\zeta}$ and $\nabla \cdot \boldsymbol{u}$ from Eq. (9), we obtain (omitting the spurious factor $(n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2 + (\boldsymbol{k} \cdot \boldsymbol{V}_0)^2$)

$$\begin{aligned} &[\{(n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2 + c^2 k^2\} \{(n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2 + (\boldsymbol{k} \cdot \boldsymbol{V}_0)^2\} + (n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2 (\boldsymbol{k} \times \boldsymbol{V}_0)^2 - (n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^4] D^2 w \\ &g D w \ (n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2 k^2 - [\{(n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2 + c^2 k^2\} \cdot \{(n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2 \\ &+ (\boldsymbol{k} \cdot \boldsymbol{V}_0)^2\} + (n + i \boldsymbol{k} \cdot \boldsymbol{U}_0)^2 (\boldsymbol{k} \times \boldsymbol{V}_0)^2] k^2 w = 0 , \end{aligned} \quad (10)$$

as the equation determining $w$.

## 3. Boundary Conditions and Dispersion Relation

For a configuration of two superposed uniform plasmas slipping past each other at the horizontal interface $z = 0$, the respective solutions, vanishing at $z = \pm \infty$, of Eq. (10) are written as:

$$w_1 = A_1 \exp\{m_1 z\} (z < 0), \qquad w_2 = A_2 \exp\{-m_2 z\} \ (z > 0) , \quad (11)$$

where

$$\begin{aligned} m_j = &\frac{1}{2[(n_j^2 + c_j^2 k^2) \{n_j^2 + (\boldsymbol{k} \cdot \boldsymbol{V}_j)^2\} + n_j^2 (\boldsymbol{k} \times \boldsymbol{V}_j)^2 - n_j^4]} \\ &\cdot [- n_j^2 k^2 g \pm \{n_j^4 k^4 g^2 + 4 k^2 [(n_j^2 + c_j^2 k^2) \{n_j^2 + (\boldsymbol{k} \cdot \boldsymbol{V}_j)^2\} + n_j^2 (\boldsymbol{k} \times \boldsymbol{V}_j)^2 - n_j^4] \\ &\cdot [(n_j^2 + c_j^2 k^2) \{n_j^2 + (\boldsymbol{k} \cdot \boldsymbol{V}_j)^2\} + n_j^2 (\boldsymbol{k} \times \boldsymbol{V}_j)^2]\}^{1/2}], \quad j = 1, 2, \end{aligned} \quad (12)$$

where $n_j = (n + i \boldsymbol{k} \cdot \boldsymbol{U}_j)$.

At the perturbed interface ($z=0$), the following boundary conditions must be satisfied.

1. The normal component of velocity is continuous, this condition leads to

$$w_1 - U_{1x}\frac{\partial\xi}{\partial x} - U_{1y}\frac{\partial\xi}{\partial y} = w_2 - U_{2x}\frac{\partial\xi}{\partial x} - U_{2y}\frac{\partial\xi}{\partial y} = \frac{\partial\xi}{\partial t}, \tag{13}$$

where $\xi$ denotes the small displacement of the interface. Using (11) we obtain

$$A_2 = (n_2/n_1)A_1. \tag{14}$$

2. The normal component of the magnetic field is continuous. This conditions is automatically satisfied as a consequence of condition 1.
3. The normal stress should be continuous across the interface.

$$\delta p_1 - \delta p_2 + (\boldsymbol{H}_1\cdot\boldsymbol{h}_1 - \boldsymbol{H}_2\cdot\boldsymbol{h}_2) = 0. \tag{15}$$

Using the above mentioned boundary conditions, we obtain the dispersion relation as

$$\varrho_1 m_1[m_1^2 + (\boldsymbol{k}\cdot\boldsymbol{V}_1)^2]\frac{n_1^2V_1^2 + c_1^2[n_1^2 + (\boldsymbol{k}\cdot\boldsymbol{V}_1)^2]}{(n_1^2 + c_1^2k^2)[n_1^2 + (\boldsymbol{k}\cdot\boldsymbol{V}_1)^2] + n_1^2(\boldsymbol{k}\times\boldsymbol{V}_1)^2} + \varrho_2 m_2[n_2^2 + (\boldsymbol{k}\cdot\boldsymbol{V}_2)^2]\frac{n_2^2V_2^2 + c_2^2[n_2^2 + (\boldsymbol{k}\cdot\boldsymbol{V}_2)^2]}{(n_2^2 + c_2^2k^2)[n_2^2 + (\boldsymbol{k}\cdot\boldsymbol{V}_2)^2] + n_2^2(\boldsymbol{k}\times\boldsymbol{V}_2)^2} = 0. \tag{16}$$

Retaining only the positive value of $m_j$, as the negative value renders both $w_1$ and $w_2$ infinite, Eqs. (12) and (16) constitute the characteristic equation for $n$.

## 4. Discussion

*Part A)* Propagation transverse to the direction of streaming ($k_x=0$, $k_y=k$):
The dispersion relation (16) in this case reduces to

$$\frac{[g^2+4(c_1^2+V_1^2)\{n^2+k^2(c_1^2+V_1^2)\}]^{1/2}-g}{n^2+k^2(c_1^2+V_1^2)} + \frac{[g^2+4(c_2^2+V_2^2)\{n^2+k^2(c_2^2+V_2^2)\}]^{1/2}-g}{n^2+k^2(c_2^2+V_2^2)} = 0, \tag{17}$$

where it is assumed that $\varrho_1=\varrho_2$.

Case 1: The streaming plasmas do not carry the same magnetic field i.e. $\boldsymbol{H}_1 \neq \boldsymbol{H}_2 (\boldsymbol{V}_1 \neq \boldsymbol{V}_2)$ and are not necessarily characterized by the same $\varkappa$ $\varkappa$ = ratio of the two specific heats $= C_p/C_v$).

(a) $\varkappa_1 \neq \varkappa_2$ (i) $c_1 \neq c_2$, (ii) $c_1 = c_2$; (b) $\varkappa_1 = \varkappa_2$, $c_1 = c_2$.

This reduces the characteristic Eq. (17) into

$$2\,\theta_1\theta_2[2(p_1\theta_2+p_2\theta_1) + (g^2+4p_1\theta_1)^{1/2}(g^2+4p_2\theta_2)^{1/2}-g^2]=0, \tag{18}$$

where $p_1=c_1^2+V_1^2$, $p_2=c_2^2+V_2^2$ and

$$\theta_1 = n^2+k^2(c_1^2+V_1^2),\quad \theta_2=n^2+k^2(c_2^2+V_2^2). \tag{19}$$

Now from Eq. (18),
either

$\theta_1=0$ i.e. $U_p^2 = -n^2/k^2 = c_1^2+V_1^2\,(+ve)$ i.e. $U_p$ is real,

$\theta_2=0$ i.e. $U_p^2 = -n^2/k^2 = c_2^2+V_2^2\,(+ve)$ i.e. $U_p$ is real, (20)

or

$$2(p_1\theta_2+p_2\theta_1)+(g^2+4p_1\theta_1)^{1/2}\cdot(g^2+4p_2\theta_2)^{1/2}-g^2=0,$$

which on simplification gives

$$(p_1-p_2)^2n^4-2g^2(p_1+p_2)n^2-g^2k^2(p_1+p_2)^2=0. \tag{21}$$

Solving for $n$ yields

$$n^2=\frac{g(p_1+p_2)}{(p_1-p_2)^2}[g\pm\{g^2+k^2(p_1-p_2)^2\}^{1/2}]. \tag{22}$$

From Eq. (22), for the positive sign within the brackets, $n^2$ is always positive i.e. at least one value of $n$ is positive. Consequently the interface is unstable. However, in the absence of gravity it reduces to the marginal state.

Case 2: (a) The streaming plasmas carry the same magnetic field and are not characterized by the same $\varkappa$ i. e.

$\mathbf{H}_1 = \mathbf{H}_2 (\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{v}$ say) and $\varkappa_1 \neq \varkappa_2$, we have

$$\theta_1 = 0 \text{ which gives } U_p{}^2 = c_1{}^2 + v^2 \text{ i. e. } U_p \text{ is real,}$$
$$\theta_2 = 0 \text{ which gives } U_p{}^2 = c_2{}^2 + v^2 \text{ i. e. } U_p \text{ is real.} \quad (23)$$

and from Eq. (22), at least one value of $n$ is positive. Hence the interface is unstable.

(b) The streaming plasmas carry the same magnetic field and are characterized by the same $\varkappa$, $\mathbf{H}_1 = \mathbf{H}_2 (\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{v}$ say) and $\varkappa_1 = \varkappa_2$, we have

$$\theta_1 = 0 \text{ which gives } U_p{}^2 = c^2 + v^2 \text{ i. e. } U_p \text{ is real,}$$
$$\theta_2 = 0 \text{ which gives } U_p{}^2 = c^2 + v^2 \text{ i. e. } U_p \text{ is real,} \quad (24)$$

and from Eq. (21),

$$U_p{}^2 = -\frac{n^2}{k^2} = \frac{(p_1 + p_2)^2}{2(p_1 + p_2)}$$
$$= p_1 [\text{Since } p_1 = p_2 \text{ from Eq. (19)}]$$
$$= c^2 + v^2 \text{ i. e. } U_p \text{ is real.}$$

Since $U_p$ always remains real, the interface is stable.

*Part B)* Propagation parallel to the direction ofreaming ($k_x = k$, $k_y = 0$):

The characteristic equation in this case becomes

$$\frac{[A_1{}^4 g^2 + 4k^2 (c_1{}^2 - A_1{}^2)(V_1{}^2 - A_1{}^2)\{(c_1{}^2 - A_1{}^2)(V_1{}^2 - A_1{}^2) - A_1{}^4\}]^{1/2} + A_1{}^2 g}{c_1{}^2 - A_1{}^2} + \frac{[B_1{}^4 g^2 + 4k^2 (c_2{}^2 - B_1{}^2)(V_2{}^2 - B_1{}^2)\{(c_2{}^2 - B_1{}^2)(V_2{}^2 - B_2{}^2) - B_1{}^4\}]^{1/2} + B_1{}^2 g}{c_2{}^2 - B_1{}^2} = 0, \quad (25)$$

where $A_1 = U_1 - U_p$, $B_1 = U_2 - U_p$.

If we take $U = -U_2 = +U_1$, the Eq. (25) reduces to

$$\frac{[A^4 g^2 + 4k^2 (c_1{}^2 - A^2)(V_1{}^2 - A^2)\{(c_1{}^2 - A^2)(V_1{}^2 - A^2) - A^4\}]^{1/2} + A^2 g}{c_1{}^2 - A^2} + \frac{[B^4 g^2 + 4k^2 (c_2{}^2 - B^2)(V_2{}^2 - B^2)\{(c_2{}^2 - B^2)(V_2{}^2 - B^2) - B^4\}]^{1/2} + B^2 g}{c_2{}^2 - B^2} = 0, \quad (26)$$

where $$A = U - U_p \text{ and } B = U + U_p. \quad (27)$$

Case 1: The streaming plasmas do not carry the same magnetic field and are not necessarily characterized by the same $\varkappa$, then the characteristic Eq. (26) becomes

$$2 M_1 M_2 [2k^2 \{M_2 N_1 (M_1 N_1 - A^4) + M_1 N_2 (M_2 N_2 - B^4)\} - g^2 A^2 B^2 + \{A^4 g^2 + 4k^2 M_1 N_1 (M_1 N_1 - A^4)\}^{1/2} \{B^4 g^2 + 4k^2 M_2 N_2 (M_2 N_2 - B^4)\}^{1/2}] = 0, \quad (28)$$

where $$M_1 = c_1{}^2 - A_2, \quad N_1 = V_1{}^2 - A^2, \quad M_2 = c_2{}^2 - B^2 \quad \text{and} \quad N_2 = V_2{}^2 - B^2. \quad (29)$$

Now from Eq. (28),

either
$$M_1 = 0 \text{ i. e. } A = \pm c_1 \text{ or } U_p = U \mp c_1$$
$$\text{both real} \quad (30)$$
$$M_2 = 0 \text{ i. e. } B = \pm c_2 \text{ or } U_p = U + c_2$$

or

$$k^2 [M_2 N_1 (M_1 N_1 - A^4) - M_1 N_2 (M_2 N_2 - B^4)]^2 = g (M_1 B^2 + M_2 A^2) [N_1 B^2 (M_1 N_1 - A^4) + N_2 A^2 (M_2 N_2 - B^4)], \quad (31)$$

Substitution of $A$, $B$, $M_1$, $N_1$, $N_2$ and $M_2$ from Eqs. (27) and (29) in Eq. (31) yields a twelveth degree equation in phase velocity.

Case 2: Let $\mathbf{H}_1 = \mathbf{H}_2 (\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{v}$ say) and $c_1 = c_2 = c$ say, then Eq. (31) becomes

$$
\begin{aligned}
&[4\,g^2(c^2+v^2)+16\,k^2\,U^2\,(c^2+v^2)^2]\,U_p^{10}-[4\,g^2\,\{c^2(c^2+v^2)+v^2\,(2\,c^2+v^2)+3\,U^2\,(c^2+v^2)\}\\
&\quad+64\,k^2\,U^2(c^2+U^2)\,(c^2+V^2)^2]\,U_p^{8}+[4\,g^2\{2\,c^2\,v^2(c^2+v^2)+4\,U^2\,v^2(2\,c^2+v^2)+2\,U^4(c^2+v^2)\}\\
&\quad+16\,k^2\,U^2\{(4\,c^4+4\,c^2\,U^2+6\,U^4)\,(c^2+v^2)^2+4\,c^4\,v^2(c^2+v^2)\}]\,U_p^{6}\\
&\quad+[8\,(c^2+v^2)\{g^2+8\,k^2\,c^2\,(c^2+v^2)\}\,U^6+\{128\,k^2\,c^6(c^2+v^2)\\
&\quad-8\,g^2(5\,c^2\,v^2+3\,v^4-c^4)\}\,U^4-8\,c^2(c^2+v^2)\{g^2\,v^2+8\,k^2\,c^4(c^2+v^2)+16\,k^2\,c^4\,v^2\}\,U^2-4\,g^2\,c^4\,v^4]\,U_p^{4}\\
&\quad+[16\,k^2(c^2+v^2)^2\,U^{10}-4\,(c^2+v^2)\\
&\quad\cdot\{3\,g^2+16\,k^2\,c^2\,(c^2+v^2)\}\,U^8\,\{64\,k^2\,c^4(c^2+v^2)\,(c^2+2\,V^2)+16\,v^2\,(2\,c^2+v^2)g^2\}\,U^6\\
&\quad-8\,c^2\,v^2(c^2+v^2)\,(g^2+16\,k^2\,c^4)\,U^4+8\,c^4\,v^4\,(8\,k^2\,c^4-g^2)\,U^2]\,U_p^{2}+[\,(c^2+v^2)\,U^{10}\\
&\quad-(c^4+3\,c^2\,v^2+v^4)\,U^8+2\,c^2\,v^2(c^2+v^2)\,U^6-c^4\,v^4\,U^4]\,4\,g^2=0\,.
\end{aligned}
\tag{32}
$$

The interplanetary plasma flow measured by Explorer 10 is supersonic in the sense that the flow speed is greater than the Alfvén speed (Kellogg [12]). Observations (Gringauz et al. [13]; Bridge et al. [14]) show that the solar wind has a speed $\cong 250-400$ km/sec. When the sun is active this value may mount to $\cong 10^3$ km/sec. The speed of the incoming solar wind is therefore supersonic as it hits the magnetosphere. Equation (32) is an equation of fifth degree in $U_p^2$ and since it is of odd degree, it must have a root of opposite sign to the last term. This last term is positive when we consider $U>V$ and $U>c$ for the solar wind and the magneto-pause. Therefore Eq. (32) has at least one negative root. From this we conclude that at least one value of $U_p$ is imaginary. Hence the interface is unstable. Chang and Russel [15] have established that the supersonic flow $(U>c)$ with gravitation in the absence of magnetic field is stable for perturbations parallel to the streaming velocity and the subsonic flow $(U<c)$ may be stabilized if the product of gravitation and surface tension is sufficiently large. In the absence of gravitation, $U=V$ is the critical limit for transition from stability to instability. Our conclusion that the supersonic flow $U>V$, $U>c$ is unstable even in the presence of gravitation, is not very surprising.

The subsonic flow when $U<V$, $U<c$ is of some theoretical interest though not applicable to the magnetopause. In this case the last term of Eq. (32) is negative. By putting $U_p^2=-U_p^2$ in Eq. (32), we find that there is no change of sign, implying thereby that there can not be any negative root. Hence we conclude that the subsonic flow in the presence of gravitation and magnetic field is stable for $\bar{k}\parallel\bar{U}$.

We conclude with the remark that our analysis of the Kelvin-Helmholtz instability including gravity may be applicable to the understanding of the stability of the magnetopause and solar wind boundary. We observe that the interface between the two compressible fluids is unstable to disturbances transverse to the streaming due to the gravity [Eq. (22)] when the streaming plasmas have different magnetic fields. When the streaming plasmas carry the same magnetic field but have different $\varkappa$, the interface is unstable while the interface with the same $\varkappa$ is found to be stable. For disturbances parallel to the streaming we find that the interface is unstable for both $\boldsymbol{H}_1\neq\boldsymbol{H}_2$ and $\boldsymbol{H}_1=\boldsymbol{H}_2$. The subsonic flow is found to be stable.

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